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Palaeomagnetic secular variation as a function of intensity

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We seek to establish whether or not secular variation, the rate at which the magnetic field is changing in time, is a function of field strength. Towards that end we examine a database consisting of palaeomagnetic directions and absolute intensities from piles of extruded lava flows, many of which record polarity transitions. We find that directions from stratigraphically adjacent lava flows are most (least) correlated when the local field strength is high (low). Since volcanic activity is unrelated to, and therefore uncorrelated with, magnetic secular variation, this relationship between angular correlation and intensity indicates that angular secular variation is quiet (enhanced) when and where the local field strength is high (low). Our conclusion is consistent with some aspects of the recent behaviour of the modern field and is qualitatively consistent with sedimentary data recording reversals. Although we find a simple relationship between angular difference and intensity, a corresponding relationship for relative intensity differences has proved to be more elusive; its possible resolution will benefit from the continued collection of full vectorial palaeomagnetic data from lavas. Statistical models of secular variation need to incorporate the information content of serially correlated stratigraphically ordered data if the lava data are to be fully exploited. We suggest that the apparent inverse relationship between angular secular variation and local field strength could be the result of electromagnetic coupling between the solid inner core and the liquid outer core, with the inner core tending to stabilize core convection, and hence the field, when the intensity is high (as has been hypothesized).

Keywords: geodynamo; geomagnetic secular variation; geomagnetism; palaeomagnetism

1. Introduction

At the Earth's various centres of volcanic activity, a history of the geomagnetic field is preserved in piles of frozen lava flows. This palaeomagnetic record is a challenge to read, but progress is being made in extracting reliable measures of the full magnetic vector, both directions and intensities. Such data have recently become rather more numerous, revealing the dynamic nature of the Earth's interior: convective motion in the outer core sustains the main part of the geomagnetic field via dynamo action; advective amplification balances diffusive destruction of the field. This motion is time dependent and thus the observed field exhibits secular variation. Since the Earth's magnetic field is spatially complex, it is not a perfect dipole, and since it exhibits a wide range of time-dependent variation, most notably undergoing occasional excursions and reversals, we would ideally like to analyse a palaeomagnetic database consisting of measurements representing a temporally dense sampling of the field taken from a geographically wide distribution of sites. Yet volcanoes are not evenly distributed over the Earth's surface and their eruptions occur sporadically, and this, together with the fact that scientists tend to sample preferentially transitional periods, means that the spatial and temporal distribution of volcanic palaeomagnetic data is far from uniform and might even be considered to be biased. Nonetheless, because palaeomagnetism provides us with the only long-term record of the geodynamo's secular variation, it is imperative that these problems be addressed when making interpretations.

By secular variation we mean the rate at which the magnetic field $\mathbf{B}$ is changing, namely $\partial_t \mathbf{B}$. This definition (Courtillot & Valet 1995) applies to all variation of internal origin regardless of its characteristic time-scale; excursions and reversals are considered to be part of the secular variation. Moreover, we apply this definition to the total field; we make no distinction between the variation (say) of the dipolar and non-dipolar fields. Our approach here is somewhat different from that conducted by some in the palaeomagnetic community, where different spatial and temporal components are considered separately. Sometimes the dipole is treated differently from other harmonics (see, for example, Hulot & LeMouël 1994) and secular variation is often defined as being separate from reversals and excursions (see, for example, Merrill & McFadden 1990). Such decompositions may at times be convenient, but since the field is complex, neat and tidy divisions are rather arbitrary and often physically impossible. In the spatial domain, the dipolar field is coupled to the non-dipolar field through the induction equation (Bullard & Gellman 1954). In the temporal domain, there is no compelling theoretical justification to draw a fine distinction between transitions and other periods of enhanced secular variation, and since palaeomagnetic data are limited in number and quality, they are often insufficient for making such distinctions. Having said all of this, we recognize that secular variation is itself variable. Indeed, we seek to measure the variation of the secular variation, but without resorting to arbitrary definitions of what is and what is not a reversal, excursion or non-transitional secular variation.

To quantify the behaviour of the Earth's magnetic field, a number of statistical studies of palaeomagnetic lava data have been conducted (Cox 1970; McElhinny & Merrill 1975; Constable & Parker 1988; Camps & Prévot 1996), usually examining the dispersion of directions in space and concentrating on the degree to which the field deviates from an axial dipole. Although directional dispersion is a manifestation of secular variation, by itself it does not tell us much about the rate at which the field changes. Moreover, many statistical studies have concentrated on non-transitional periods, when the field intensity is highest. Although the field spends most of its time in a nearly dipolar non-transitional (reverse or normal) state, it does not necessarily mean that when the field deviates from an axial dipole, or suffers a diminution in intensity and undergoes a transition, that the field then changes at an enhanced rate, though this may in fact be the case.

Information about secular variation is contained in temporal correlations between lava data, yet we know of no attempt to incorporate such information into statistical models; the data are usually treated as independent quantities, a shortcoming which has at least been recognized by some (Constable 1990; Hulot & LeMouël 1994). This practice stems, in large part, from difficulty in establishing a time-scale; volcanic eruptions are highly sporadic and lavas are not easily dated radiometrically. But partial knowledge of temporal variation can come from stratigraphy: the data are temporally ordered if they are gathered from piles of serially deposited lava flows. A high degree of correlation between palaeomagnetic data from stratigraphically adjacent flows is to be expected if either the flows were deposited closely in time or if the magnetic field did not change much between successive depositions; conversely, a low degree of correlation is to be expected if either the duration between successive depositions was long or if the field was changing relatively rapidly between depositions. Given just a few data, without independent knowledge of depositional dates there is no objective means of untangling these two effects. Fortunately, volcanic activity is unrelated to, and therefore uncorrelated with, variations in the magnetic field. Thus, provided enough data are analysed, it should be possible to incorporate the mean of correlations between serially ordered data into statistical measures of secular variation, since the durations between consecutive depositions, although by no means uniformly distributed in time, are at least unbiased with respect to geomagnetic activity. Without some accounting for temporal correlation the data remain underexploited.

In response to recent interest in absolute intensity measurements from lavas, here we inspect these data with the aim of describing the statistics of secular variation. In particular, we are curious as to whether or not mean secular variation depends on field intensity. Partial inspiration for our investigation comes from inspection of the geographical distribution of modern field variation. In figure 1 we show a map of the Earth's mean surficial field intensity, as well as different parts of the secular variation. Notice that the field is changing most rapidly in the middle of the Atlantic, which is also where the field is weakest. Of course modern data only record a rather brief period in the history of the Earth's magnetic field; it is possible that the inverse correlation seen in figure 1 between secular variation and intensity could be transient or coincidental. Moreover, even if there is a relationship between secular variation and intensity it might be complex. The present range of field intensity over the Earth's surface is relatively small, making an investigation of such a functional relationship difficult.

To study a broad range of field intensities we turn to palaeomagnetic data. For this analysis we assembled a database consisting of palaeomagnetic directions and absolute intensities from consecutively deposited lava flows. Rather than simply repeat the dispersion analyses of our predecessors, we examine correlations between vectors from stratigraphically adjacent flows. Insofar as mean correlations reflect the mean secular variation, we can investigate the rate at which the field changes as a function of local field strength, a functional dependence which can be represented as

$$\partial_t \mathbf{B} = \partial_t \mathbf{B}(\sqrt{\mathbf{B}\cdot\mathbf{B}}). \tag{1.1}$$

Our study here should be compared with that of Love (2000); from his analysis of directional lava data, he concluded that palaeosecular variation is a function of field direction, finding, consistent with sedimentary data, that secular variation is enhanced (quiet) during transitional (non-transitional) periods.

2. The database: selection criteria

Palaeomagnetic data recording a broad range of intensities are of either sedimentary or volcanic types. Sediments can yield more or less continuous records of magnetic

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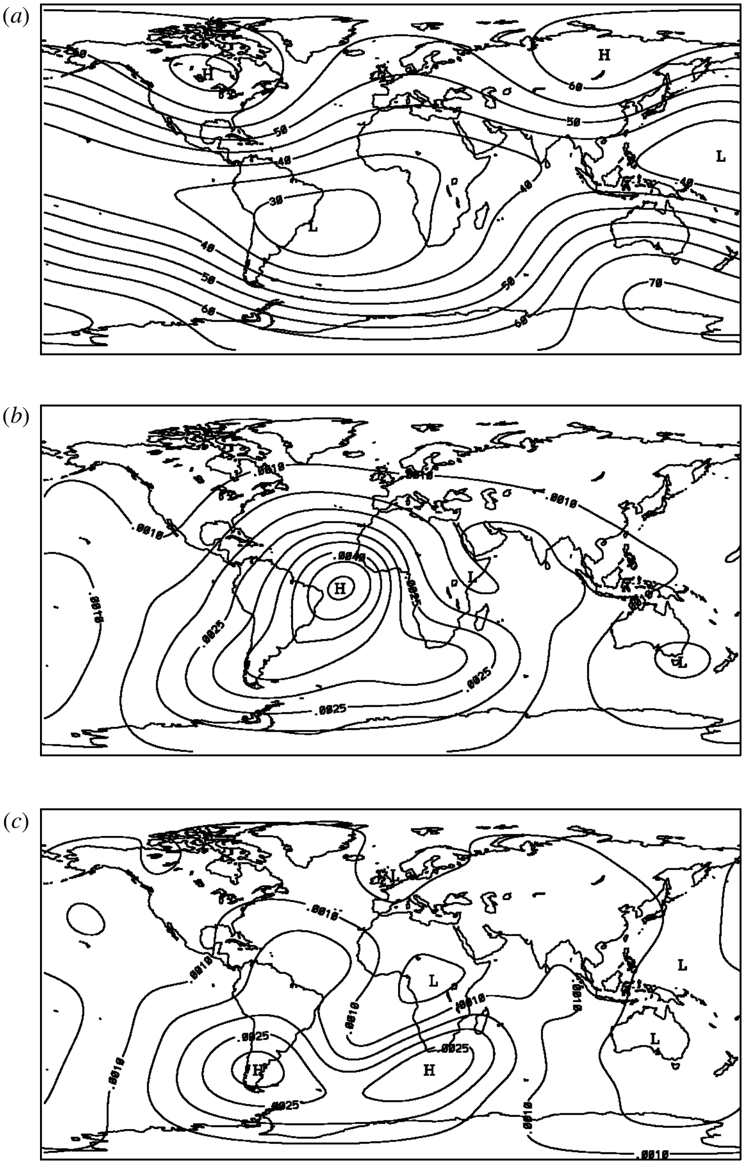

Figure 1. (*a*) The mean (1840–1990) surficial field intensity (μT) from the historical model of Bloxham & Jackson (1992); global average: 53 μT. (*b*) The average relative secular variation $\partial_t B$ (yr^{-1}), equation (4.1); global average: 0.098° per century. (*c*) The average relative intensity variation $\partial_t F$ (yr^{-1}), equation (4.3); global average: 0.061° per century.

field variation, while lava data, due to the sporadic nature of volcanic activity, give highly discontinuous records of magnetic-field variation. On the other hand, individual measurements from lavas are not subject to the controversies over reliability that make interpretation of sedimentary data so difficult (Hoffman & Slade 1986; Tauxe 1993); this is particularly true for rock deposited during periods of low field strength. Moreover, lava data can yield absolute intensity values, while sedimentary

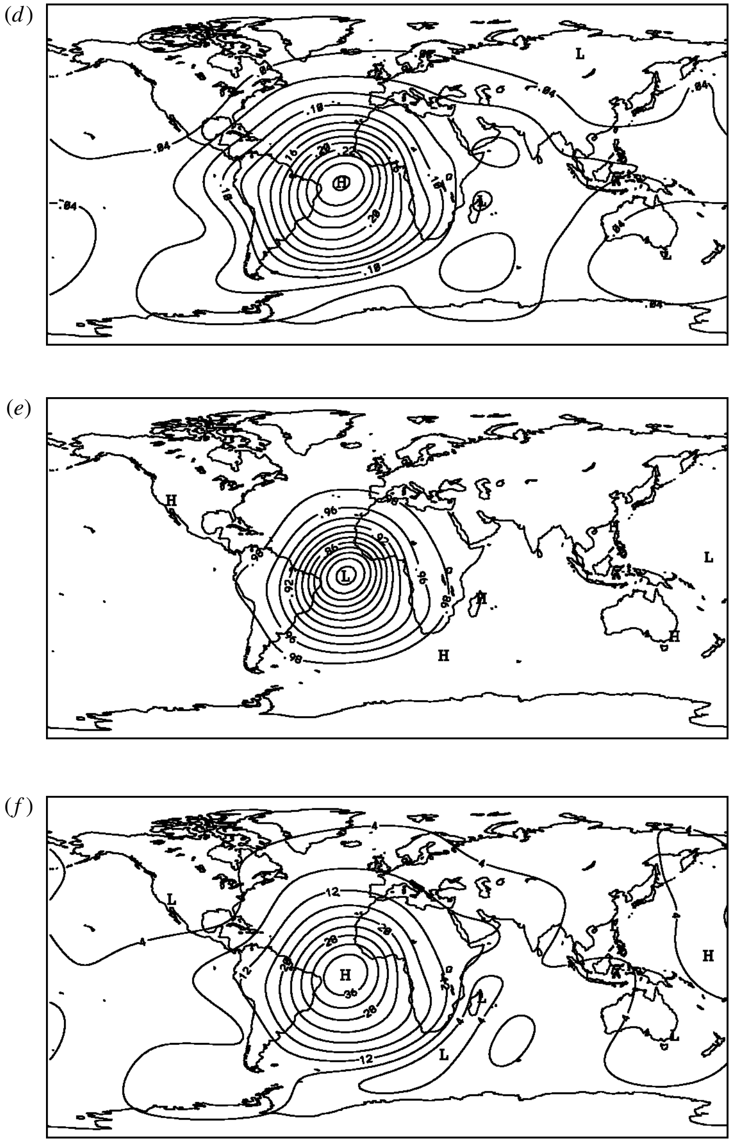

Figure 1. (*Cont.*) (*d*) The average directional variation $\partial_t A$ (deg yr^{-1}), equation (4.5); global average: 4.4° per century. (*e*) The angular correlation r between vectors at 1840 and 1990, equation (4.6); global average: 0.9956. (*f*) The angular difference ξ between vectors at 1840 and 1990, equation (4.7); global average 5.4°. Note that the total secular variation $\partial_t B$ is highest in the middle of the Atlantic, where the intensity is lowest. Most of this correlation is due to the angular secular variation $\partial_t A$; the correlation of relative intensity variation $\partial_t F$ with the intensity low is not as good. L denotes low, H denotes high.

data can only yield relative intensity variation. Despite obvious difficulties presented by temporal discontinuities, we have chosen to analyse lava data.

The data used in this study come from published studies of stratigraphically ordered extruded lava piles. Each palaeomagnetic direction in our database, incli-

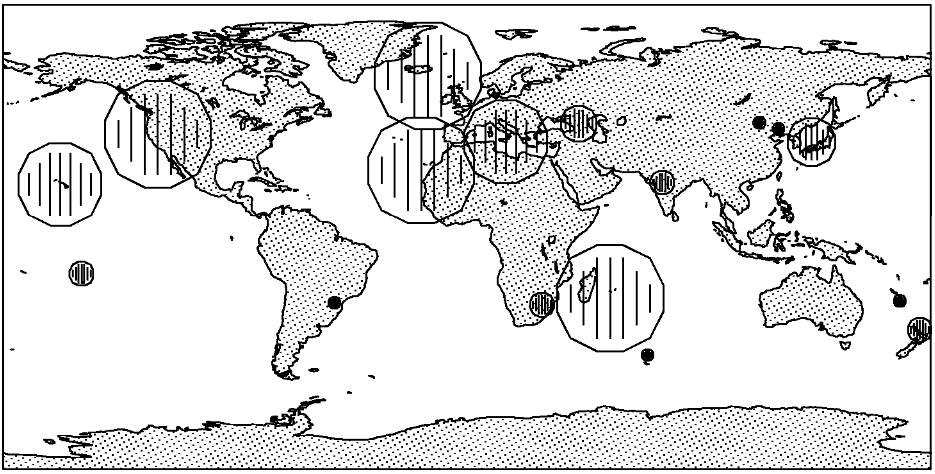

Figure 2. Geographical distribution of data sites; the size of the symbol is proportional to the number of data, absolute intensities (F) and directions (I, D) from each site.

nation and declination (I, D), is an average of measurements from at least two magnetically cleaned samples per flow, with the precision parameter α_{95}, the semiangle of the cone of 95% confidence centred on the mean direction, less than 20°; these selection criteria are similar to those of our predecessors (Camps 1994; Quidelleur *et al.* 1994; McElhinny & McFadden 1997). The (absolute) intensities (F) in our database consist of Thellier- or Shaw-type measurements, also made from at least two samples per flow. Generally speaking, the Thellier method is preferable (Prévot & Perrin 1992): Although laboratory comparisons have shown that the Shaw method usually yields individual results similar to the Thellier method (Kono 1978; Senanayake *et al.* 1982), data compilations indicate that Shaw intensities are generally somewhat more scattered than Thellier intensities (Tanaka *et al.* 1995*a*). In all source papers considered here, authors report not only the mean of multiple measurements of F but also the number of measurements N and the standard deviation of the different intensity measurements σ_F. Consistent with Student's t-distribution, the error on the mean is estimated as $\sigma(F) = \sigma_F/\sqrt{N}$. We accept only absolute intensities F where the relative error $\sigma(F)/F$ is less than 33%. On the basis of stratigraphy, the data are of known temporal order and are therefore useful for a correlation analysis. Palaeomagnetic data coming from intrusive igneous bodies (dykes) are of unknown temporal order; we do not consider such data here. Among the various sites there are 53 stratigraphic sections and 352 directions/intensities. The geographical distribution of the sample sites is shown in figure 2; the database is summarized in table 1.

3. Dispersion of the data

Before turning to our correlation analysis, we inspect the dispersion of the data, and for this we restrict ourselves to lava flows less than 20 Myr in age, a period of time over which most plate tectonic movements can be neglected relative to the 180° directional change of reversals. We measure the dispersion in three different ways. First, for intensities from different sites we normalize by the expected latitudinal intensity dependence of an axial dipole:

$$F_{\mathrm{N}} = F(1 + 3\sin^2 \Lambda)^{-1/2}, \tag{3.1}$$

where Λ is the palaeosite latitude, which for these youngish data is nearly the same as the present-site latitude. Second, for directions we consider a coordinate system fixed to the local axial dipole direction. The Cartesian components of the total magnetic field vector are

$$\mathbf{B} = (X, Y, Z) = (F\cos I \cos D, F\cos I \sin D, F\sin I); \tag{3.2}$$

while for a unit magnetic field the components are

$$\hat{\mathbf{B}} = (\hat{X}, \hat{Y}, \hat{Z}) = (\cos I \cos D, \cos I \sin D, \sin I). \tag{3.3}$$

The off-dipole angle θ is given by

$$\cos\theta = \hat{X}\cos I_{\mathrm{AD}} + \hat{Z}\sin I_{\mathrm{AD}}, \tag{3.4}$$

where the inclination for an axial dipole is given by

$$\tan I_{\mathrm{AD}} = 2\tan\Lambda. \tag{3.5}$$

Third, we also display palaeodirections in terms of virtual geomagnetic poles (VGPs), the south magnetic pole of a dipole corresponding to field directions from each palaeomagnetic site; commonly directional dispersion is expressed in terms of VGP latitude λ (Cox 1970). For our database, the population distributions of these three quantities are shown in figure 3. The directions show the familiar bimodal distribution, with the majority of directions corresponding nearly to an axial dipole (θ near 0° and 180°, λ near ±90° N). It is, however, worth recognizing that since palaeomagnetists routinely preferentially sample transitional periods, the number of intermediate directions (θ near 90°; λ near 0° N) in our database are over-representative of the Earth: 25% of the VGP latitudes fall between ±60°. The same kind of sampling bias is reflected in the relatively large number of low-intensity data.

Following Dagley & Wilson (1971), among others, we show in figure 4 the normalized field intensity F_{N} as a function of VGP latitude. Here we see the by now familiar pattern: intensity ranges over almost an order of magnitude when the VGP is near ±90° N, while during transitional periods the intensity is relatively low. Alternatively, we can plot the VGPs from our database on a map; in figure 5 we show the geographical locations of the VGPs where the size of the plotted symbol is proportional to the normalized intensity F_{N}. Again we see that low-latitude VGPs tend to correspond to low intensities, while VGPs near ±90° N can correspond to a range of intensities. Of course, the patterns seen in these figures tell us only about the state of the field at particular moments in time; they do not tell us much about the rate of secular variation. For that we need to consider collectively the temporal order of the data.

4. Measuring secular variation: correlations

We now inspect the mean correlation of the magnetic vectors across adjacent stratigraphic levels as a function of intensity, as opposed to the dispersion of the data,

Table 1. *Database summary*

(Notation: locality denotes the location of the palaeomagnetic sections; name denotes the flow names in the source papers; lat. and long. denote the present latitude and longitude of the site; N denotes the number of palaeomagnetic vectors from each section; age denotes the estimated age of the stratigraphic section; author denotes the source paper.)

locality	name	lat. ° N	long. ° E	N	age (kyr)	author[a]
Brazil, Paraná	CV02, 12;	−29.7	308.5	2	133.0	K98
	JS02, 09, 12			3		
Canary, La Palma	LL135, 134, 132, 130, 129;	28.6	342.2	5	0.8	QV96
	LS120, 118, 114, 113, 110			5		
Canary, La Palma	ET03–08, 10, 14–20;	28.6	342.2	15	0.8	V99
	ME02–12, 15, 19–24, 26, 27, 29–32, 38–42, 44, 47, 48			32		
China, Datong	SHU, SHT	40.2	113.4	2	0.2	Z90
China, Tongjing	SB, US, UD, DD	37.8	120.8	4	0.8	Z91
Georgia, Akhalkalaki	W1, 5, 11–13, 17, 19;	41.5	43.3	7	3.6	Ca96
	Y2, 4, 9, 11, 15–17			7		
Hawaii, Oahu	F69, 71, 72;	21.4	202.1	3	1.9	DD73, Co84
	O27, 28, 35, 42;			4	3.0	
	N01, 2, 4			3	3.6	
Hawaii, Hawaii	MPII, MPI	19.5	204.5	2	0.0	Co78
Hawaii, Kawai	KT5, 9, 13, 18, 23, 26, 28–30, 32;	22.1	200.4	10	3.8–5.0	BC84
	A5, 7, 16			3		
Hawaii, Hawaii	HW05, 03, PO01, HW01, 02, 04, WA01	20.1	204.3	7	0.0–0.4	B97
Iceland, Hundsvik	A34, 35	65.2	346.4	2	12.5	L70, DL74
Iceland, Neskaupstad	B7, 9, 10, 11			4	12.3	

[a] K98, Kosterov *et al.* (1998); QV96, Quidelleur & Valet (1996); V99, Valet *et al.* (1999); Z90, Zhu *et al.* (1990); Z91, Zhu *et al.* (1991); Ca96, Camps *et al.* (1996); DD73, Doell & Dalrymple (1973); Co84, Coe *et al.* (1984); Co78, Coe *et al.* (1978); BC84, Bogue & Coe (1984); B97, Brassart *et al.* (1997); L70, Lawley (1970); DL74, Dagley & Lawley (1974).

Table 1. (*Cont.*)

locality	name	lat. ° N	long. ° E	N	age (kyr)	author[a]
Iceland, Hvalfjordur	SB'1/2, 5, 14;	64.2	338.0	3	2.1	G99
	PV1–3, 5, 7A, 7B, 8, 9;			8		
	PV'1–5, 9;			6		
	HU'3/4A, HU'4A, HU4A, 4B, 5–8, 11, 12, 19, 20, 27;			13		
	FL*6, 7;			2		
	FL4, 8, 9;			3		
	KY'0, KY1–3, 7, KY'12A			6		
Iceland, Jökuldalur	GH14, 15	65.2	344.8	2	1.6	W75, S82
Iceland, Bessastadaa	EQ33, 36, 42	65.2	344.8	3	4.8	M76, S82
Iceland, Borgarfjördur	NP176, NP4	65.2	344.8	2	4.1–6.7	W77, S82
Iceland, Southwest	RK01, 04, 14, 16–18, 22–24	64.4	338.5	9	2.5	T95
India, Deccan	MA002, I, J, M, W, Z5, Z6	17.9	75.7	7	60.0	K74
Japan, Usami	UV13, 7	35.0	139.0	2	0.8	K68
Japan, Kuju	K4, 7, D4, Dh, Kp	33.1	131.2	5	Pleistocene	N69, SM69
Japan, Kyushu Kuju	Tp, P1, Rw			3	Pliocene	
Japan, Nagao-toge	HN19, 12, 9	35.0	139.0	3	Brunhes	K71
Japan, Fuji	FJ9, FJ1	35.5	138.9	2	0.0	T90
Japan, Daisen	DS02, 4	35.3	133.6	2	0.0	T94
Kerguelen	2, 4	−50.0	70.0	2	21.9	D90
Lesotho	CV02, 12	−29.6	29.0	2	180.0	K97

[a] G99, Goguitchaichvili *et al.* (1999); W75, Watkins *et al.* (1975); S82, Senanayake *et al.* (1982); W77, Watkins *et al.* (1977); M76, McDougall *et al.* (1976); T95, Tanaka *et al.* (1995*b*); K74, Kono (1974); K68, Kono (1968); N69, Nishida *et al.* (1969); SM69, Sasajima & Maenaka (1969); K71, Kono (1971); T90, Tanaka (1990); T94, Tanaka *et al.* (1994); D90, Derder *et al.* (1990); K97, Kosterov *et al.* (1997).

Table 1. (*Cont.*)

locality	name	lat. ° N	long. ° E	*N*	age (kyr)	author[a]
New Zealand, Ruapehu	NR29, 24, 18, 10;	−39.3	175.6	4	0.0–0.2	T97
	NR7, 5, 1			3		
Norfolk	6, 7;	−29.0	167.8	2	2.3–3.1	AM73, S82
	9, 10			2		
Oregon, Steens	A70, 66, 65, 61, 59–56, 54, 52, 51, 49, 48, 46, 39, 38, 32, 31, 29–25, 19–16, 14, 11, 9, 7, 6;	42.6	240.0	32	15.5	M85, P85
	B102, 92, 85, 84, 82, 80, 78–76, 74, 73, 71, 70, 67–65, 62, 54;			18		
	C83, 80			2		
Réunion	RA17, 15, 13, 11, 9, 7, 6, 4, 3, 1;	−21.1	55.5	10	0.0	C91
	RB12–9, 4, 3b, 3, 2, 1;			9		
Réunion	RE61, 59–57, 52, 51, 46–43, 39, 33, 31, 27, 23, 15–10, 9b, 7b, 7	−21.1	55.5	24	0.0	R96
Sicily, Etna	1169, 1284, 1334, 1408, 1444, 1536, 1566, 1610, 1634, 1651, 1669, 1689, 1763, 1780, 1792, 1843, 1853, 1886, 1923, 1947, 1950, 1981	37.6	15.0	22	historical	RS86
Sicily, Volcano	VU38, 36, 28–25, 21, 20;	38.0	15.0	8	0.1	L97
	VU4, 2			2		
Tahiti, Punaruu	BKB, BKI, PKC, BKK, BKP, M2C, R1N, R1U, TR-TS	−17.6	210.3	9	0.6–1.2	C90

[a] T97, Tanaka *et al.* (1997); AM73, Aziz-Ur-Rahman & McDougall (1973); S82, Senanayake *et al.* (1982); M85, Mankinen *et al.* (1985); P85, Prévot *et al.* (1985*b*); C91, Chauvin *et al.* (1991); R96, Raïs *et al.* (1996); RS86, Rolph & Shaw (1986); L97, Laj *et al.* (1997); C90, Chauvin *et al.* (1990).

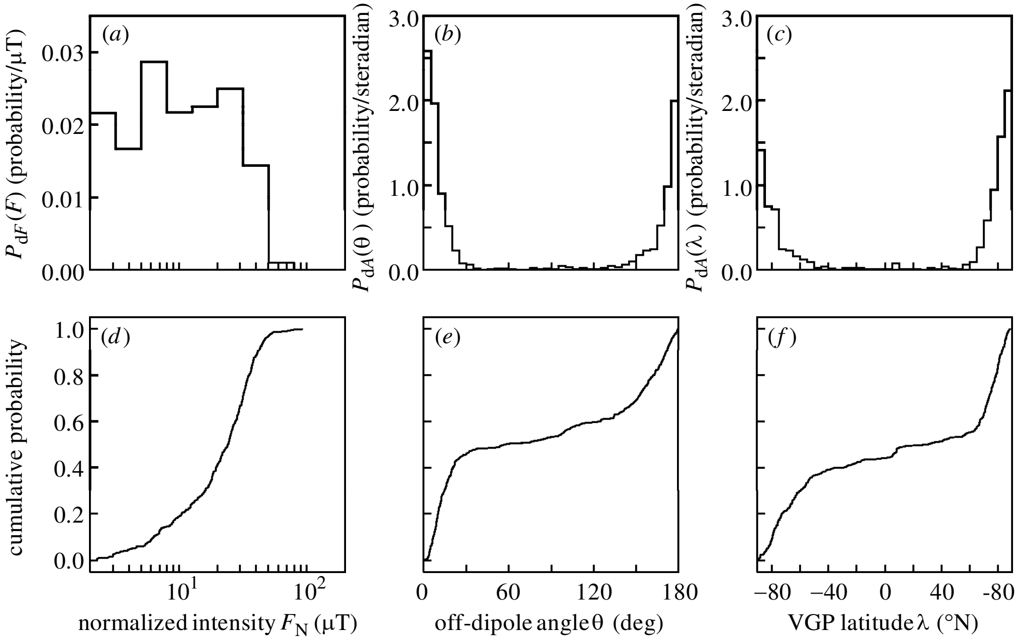

Figure 3. Probability density functions of our palaeomagnetic database: (*a*) absolute normalized intensity F_N, equation (3.1); (*b*) off-dipole angle θ, equation (3.4); (*c*) VGP latitude λ. Cumulative probability functions of our palaeomagnetic database: (*d*) absolute normalized intensity F_N; (*e*) off-dipole angle θ; (*f*) VGP latitude λ. The data show the familiar bimodal (normal/reverse) distribution in directions, but transitional directions and low intensities are somewhat over-represented in this database compared with the actual distribution of the geomagnetic field. Only data from lava flows less than 20 Myr in age are shown.

something which allows us to avoid some of the complications caused by sampling bias. We are interested in how the serial correlation, used as a measure of secular variation, depends on the local intensity across the entire spectrum of secular variation, including reversals, excursions and non-transitional periods. We do not attempt to draw a distinction between the axial dipole contribution to the intensity and the contribution made by the other multipolar constituents of the field. We note that it has often been shown that the variance of palaeodirections exhibits some latitudinal dependence (Cox 1970; McFadden *et al.* 1988; Constable & Johnson 1999), some of which is correlated with (the factor of two) latitudinal intensity dependence contributed by the axial dipole. This is a potentially important relationship, which would be removed if we normalized the intensities for site palaeolatitude, as in equation (3.1). Here, we seek to quantify secular variation in a new way: rather than study secular variation and its functional dependence on site, we study secular variation and its functional dependence on local intensity. We note that the latitudinal intensity dependence of an axial dipole is actually rather small compared with intensity variation exhibited by the geomagnetic field during reversals and excursions.

For our correlation study it is irrelevant whether or not the sampled site has undergone significant (or even unknown) tectonic movement, so long as each stratigraphic sequence was not shifted tectonically during deposition. This appears to be a reasonable assumption for many of the available studied sites, and as a result we

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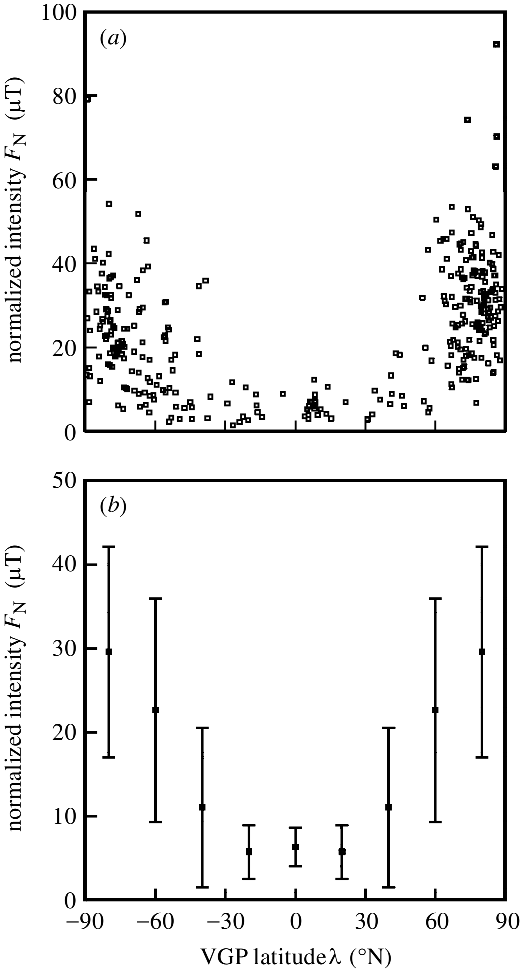

Figure 4. (*a*) Normalized intensity F_N, equation (3.1), versus VGP latitude λ. Note that transitional (non-transitional) directions correspond, on average, to low (high) intensities. Only data from lava flows less than 20 Myr in age are shown. (*b*) The mean intensity for 20° latitude bins and the standard deviation, shown with error bars; here we have assumed, consistent with the equations of magnetohydrodynamics, that the functional relationship between VGP latitude and intensity is symmetric under change in sign of the magnetic field (Merrill *et al*. 1979; Gubbins & Zhang 1993); this reduces some of the scatter in the binned means. For a given VGP latitude the intensity can vary quite a bit, but transitional (non-transitional) directions correspond, on average, to low (high) intensities. Compare with figure 5.

can consider palaeomagnetic data from some old stratigraphic sequences; the oldest record in our database is *ca.* 180 Myr in age. Some of the stratigraphic sections in our database are short, consisting of only a few palaeomagnetic measurements. It might be suggested that long stratigraphic sections are preferable, the thinking being that they (somehow) preserve a more representative record of secular variation. However, since we are concerned with the correlation between pairs of directions, we have no motivation to discard short stratigraphic sections and keep only long ones. After all,

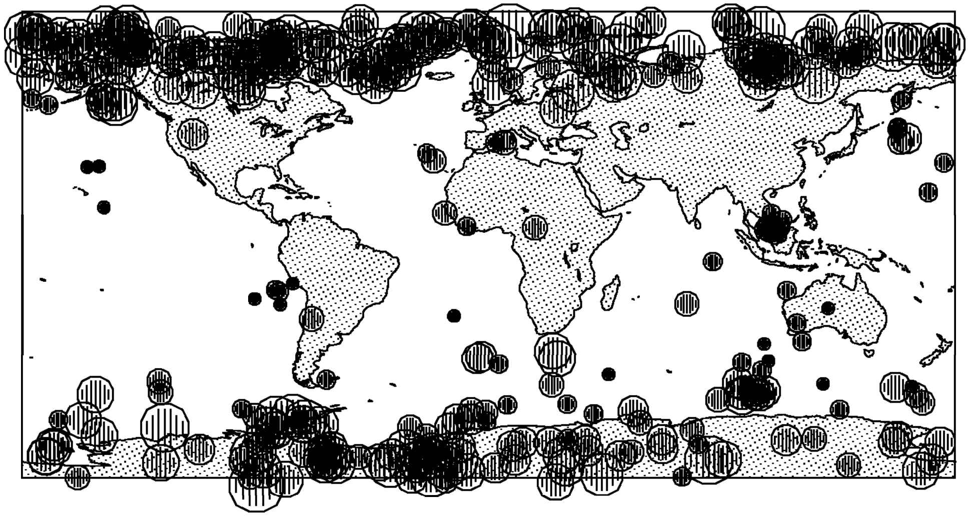

Figure 5. Map showing the location of the VGPs from our database, where the size of the symbol plotted is proportional to the normalized intensity F_N. Like figure 4, this shows that transitional (non-transitional) directions correspond, on average, to low (high) intensities. Only data from lava flows less than 20 Myr in age are shown.

two flows give one correlation which is no less interesting than individual correlations between pairs of flows from giant stacks of lava piles. Furthermore, we have no non-arbitrary means of distinguishing a 'long' stratigraphic section from a 'short' one, nor do we have any objective means of determining exactly when one stratigraphic section preserves a 'complete' record of secular variation and when another one does not. The only objective approach to a correlation study such as ours is to consider simply all of the available data which satisfy certain minimum criteria for quality, where those criteria are unrelated to the directions and correlations themselves. To do anything else would bias our analysis.

If we have temporally continuous data, one measure of the relative secular variation is given by

$$\partial_t \mathcal{B} = \left| \frac{\partial_t \mathbf{B} \cdot \partial_t \mathbf{B}}{\mathbf{B} \cdot \mathbf{B}} \right|^{1/2}. \tag{4.1}$$

Of course, the lava data in our database are discrete, thus a corresponding measure of secular variation, here called the relative secular difference, is

$$\frac{\Delta B}{B} = 2 \left| \frac{(\mathbf{B}_{j+1} - \mathbf{B}_j) \cdot (\mathbf{B}_{j+1} - \mathbf{B}_j)}{(\mathbf{B}_j + \mathbf{B}_{j+1}) \cdot (\mathbf{B}_j + \mathbf{B}_{j+1})} \right|^{1/2}, \tag{4.2}$$

where the subscript j denotes a particular lava flow and $j+1$ denotes its stratigraphic neighbour. Alternatively, we can inspect intensity and directional (angular) variation separately. For continuous data the relative variation in intensity is

$$\partial_t \mathcal{F} = |\partial_t F|/F, \tag{4.3}$$

but for discrete data the relative intensity variation is approximated by

$$\frac{\Delta F}{F} = 2\left|\frac{(\mathbf{B}_j + \mathbf{B}_{j+1}) \cdot (\mathbf{B}_{j+1} - \mathbf{B}_j)}{(\mathbf{B}_j + \mathbf{B}_{j+1}) \cdot (\mathbf{B}_j + \mathbf{B}_{j+1})}\right|. \tag{4.4}$$

Since the palaeointensity may be different from one stratigraphic level to another, equation (4.4) measures the secular variation parallel to the average vector $\frac{1}{2}(\mathbf{B}_j + \mathbf{B}_{j+1})$, as opposed to the average of the two directions $\frac{1}{2}(\hat{\mathbf{B}}_j + \hat{\mathbf{B}}_{j+1})$. For continuous data the angular variation is

$$\partial_t \mathcal{A} = [(\partial_t I)^2 + (\cos I \partial_t D)^2]^{1/2}. \tag{4.5}$$

For discrete data we can measure the angular variation in one of two ways, either by comparing the angular correlation between stratigraphically adjacent vectors (Watson & Beran 1967),

$$r = \hat{\mathbf{B}}_j \cdot \hat{\mathbf{B}}_{j+1}, \tag{4.6}$$

or by comparing the angular difference between stratigraphically adjacent vectors ξ, given by

$$\cos \xi = r. \tag{4.7}$$

Obviously, these two measures are not independent, but angular differences are somewhat easier to interpret.

The finite-difference approximations (4.2) and (4.4) will become undefined if $\mathbf{B}_j = -\mathbf{B}_{j+1}$, which could happen, for example, if we were trying to estimate relative secular difference using reverse and normal vectors. Obviously, the finite-difference measures of secular variation become meaningless if the temporal difference in depositional times is too large. For most of the analysis we consider all consecutive directions such that $\xi < \xi_c = 60°$. Given the present rate of angular secular variation, this cut-off amounts (roughly) to durations less than about 1000 years. Such a time-scale is shorter than the duration of most reversals and is comparable with the time-scale for a convective overturn in the core, which might be considered appropriate for studies of secular variation, although obviously secular variation occurs over a wide variety of time-scales (Courtillot & LeMouël 1988) and as such this estimated time-scale should not be overemphasized. Near the end of our discussion, we will consider the effects of different cut-offs of ξ.

If the palaeomagnetic data are serially correlated, which is what we expect if successive lava depositions occur closely in time, then the mean correlation $\langle r \rangle$ of the data should exceed that for a random and uniform (isotropic) distribution of directions. The expected value of ξ for a uniform distribution of directions falling within a cut-off of ξ_c is obtained by integration of ξ over a spherical cap of angular radius ξ_c:

$$\bar{\xi}(\xi_c) = \frac{\displaystyle\int_0^{\xi_c} \varepsilon \sin \varepsilon \, d\varepsilon}{\displaystyle\int_0^{\xi_c} \sin \varepsilon \, d\varepsilon} = \frac{\sin \xi_c - \xi_c \cos \xi_c}{1 - \cos \xi_c}. \tag{4.8}$$

The corresponding angular correlation is

$$\cos \bar{\xi} = \bar{r}. \tag{4.9}$$

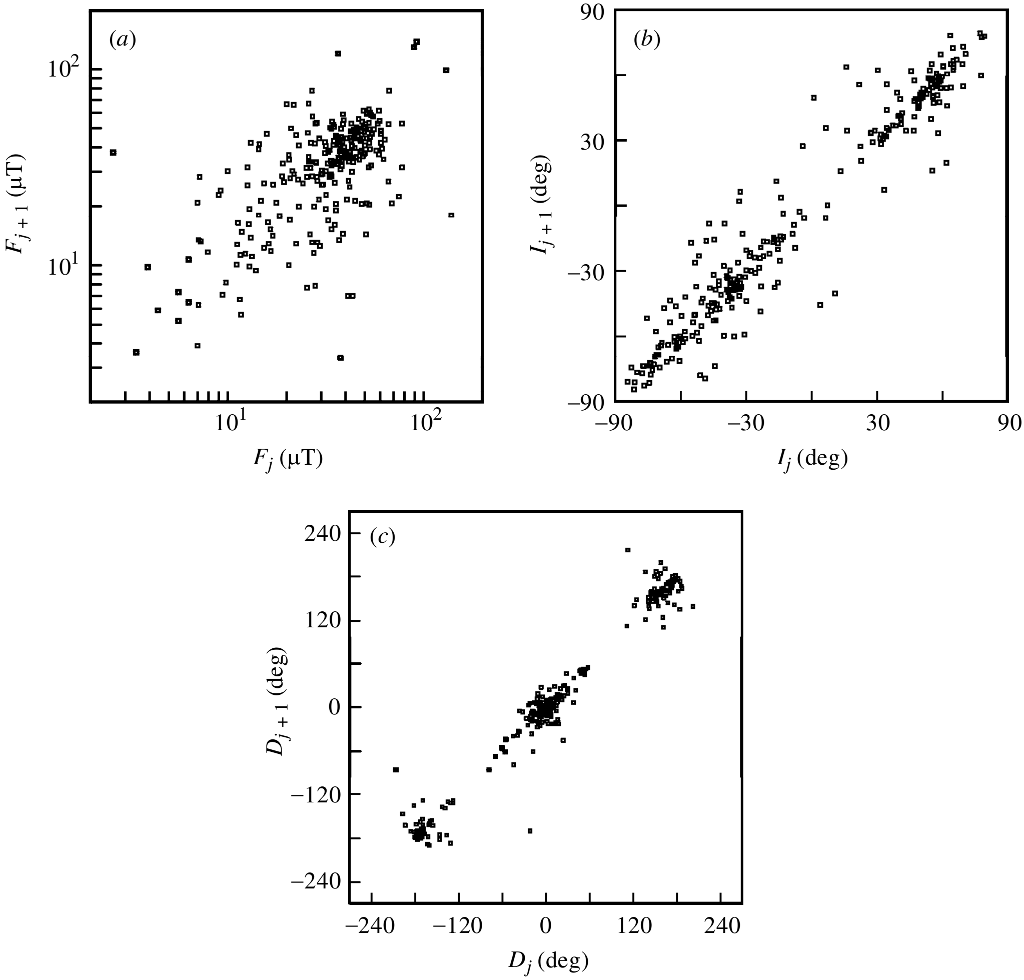

Figure 6. Serial correlation of the palaeomagnetic data from stratigraphically neighbouring lava flows: (*a*) absolute intensities F; (*b*) inclinations I; (*c*) declinations D.

Average correlations in the data can be deemed to be statistically significant with a certain degree of confidence if the correlation minus the confidence limit exceeds the expected correlation $\bar{r}$, or, equivalently, if the mean angular difference plus the confidence limit is less than the expected angular difference $\bar{\xi}$.

5. Distributions of the correlations

Before considering the various measures of secular variation as a function of local intensity, in figure 6 we show the serial correlation of the data themselves from stratigraphically adjacent flows. Note that the absolute intensities (F) and the directions (I, D) show significant serial correlation; this is true during both normal and reverse periods (the declinations are concentrated at 0° and ±180°), and there is also even some hint of serial correlation during transitional periods. These results might indicate that the data are sufficient to resolve some aspects of secular variation, though

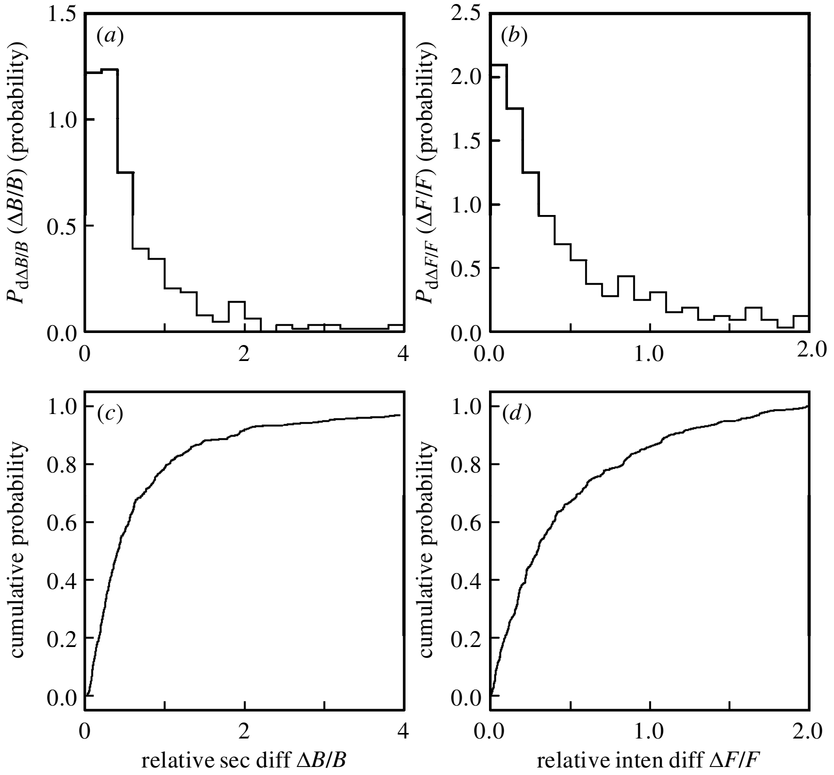

Figure 7. Probability density functions from our palaeomagnetic database: (*a*) relative secular difference $\Delta B/B$, equation (4.2); (*b*) relative intensity difference $\Delta F/F$, equation (4.4). Cumulative probability functions from our palaeomagnetic database: (*c*) relative secular difference $\Delta B/B$; (*d*) relative intensity difference $\Delta F/F$. Note that most of the relative secular differences and relative intensity differences are rather small, indicating a high degree of correlation among the majority of the data.

as we have remarked, a high degree of correlation between palaeomagnetic data from stratigraphically adjacent flows is to be expected if either the flows were deposited closely in time or if the magnetic field did not change much between successive depositions; the off-diagonal scatter seen in figure 6 could be due to either long intervening periods of time or the field changing relatively rapidly between depositions. This figure by itself offers no clue as to the relative importance of these two effects.

Since volcanic activity tends to occur sporadically, we expect that the elapsed time between successive depositions within a single lava pile is almost always relatively short compared with the time-scales characterizing secular variation, or the duration (say) of a polarity transition. In figures 7 and 8 we show the population distributions of the different measures of correlation. Concerning the relative secular difference $\Delta B/B$ and relative intensity difference $\Delta F/F$ from our database, from the cumulative distributions we see that *ca.* 80% of the relative secular differences are less than about 1, and that *ca.* 85% of the relative intensity differences are less than about 1. Concerning the angular correlations r and angular differences ξ, we see that the data

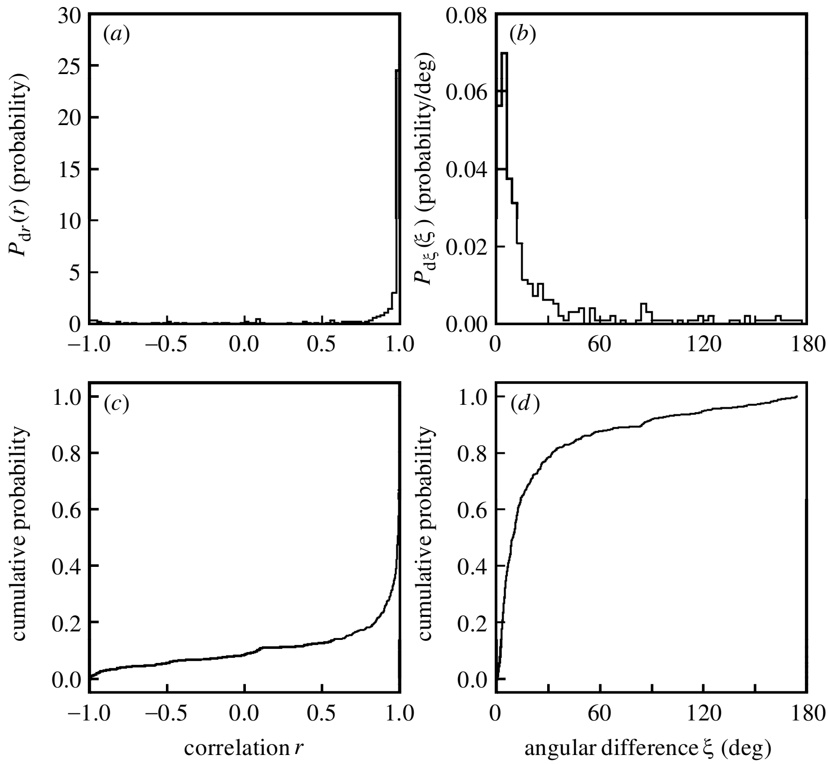

Figure 8. Probability density functions from our palaeomagnetic database: (*a*) angular correlation r, equation (4.6); (*b*) angular difference ξ, equation (4.7). Cumulative probability functions from our palaeomagnetic database: (*c*) angular correlation r; (*d*) angular difference ξ. Note that most of the correlations are high, most of the angular differences are small, both indicating a high degree of correlation amongst the majority of the data.

tend to be highly correlated, almost all of the angular correlations are greater than 0.9; and *ca.* 85% of the angular differences are less than our preferred cut-off of 60°.

Within a lava pile, each pair of flows represents a certain duration of time. Without radiometric dating these individual durations remain unknown, but on average one might expect that small (large) differences in stratigraphic position represent short (long) periods of time, during which the field will have changed by a certain amount and the correlations between palaeomagnetic vectors will be good (poor). For our database, in figures 9 and 10 we plot the various measures of correlation, both without any cut-off and with our preferred cut-off of 60°, all as a function of difference in stratigraphic position (height). Note that the average correlations decrease (increase) with an increase (decrease) in stratigraphic positional difference; this simple relationship is somewhat more distinct when a 60° cut-off is applied.

6. Variation of the secular variation

Prior to inspecting the palaeomagnetic secular variation, it is worthwhile reinspecting some of the characteristics of the modern secular variation, which has been accurately recorded for about 150 years. Although the mean secular variation $\partial_t\mathcal{B}$ is geographi-

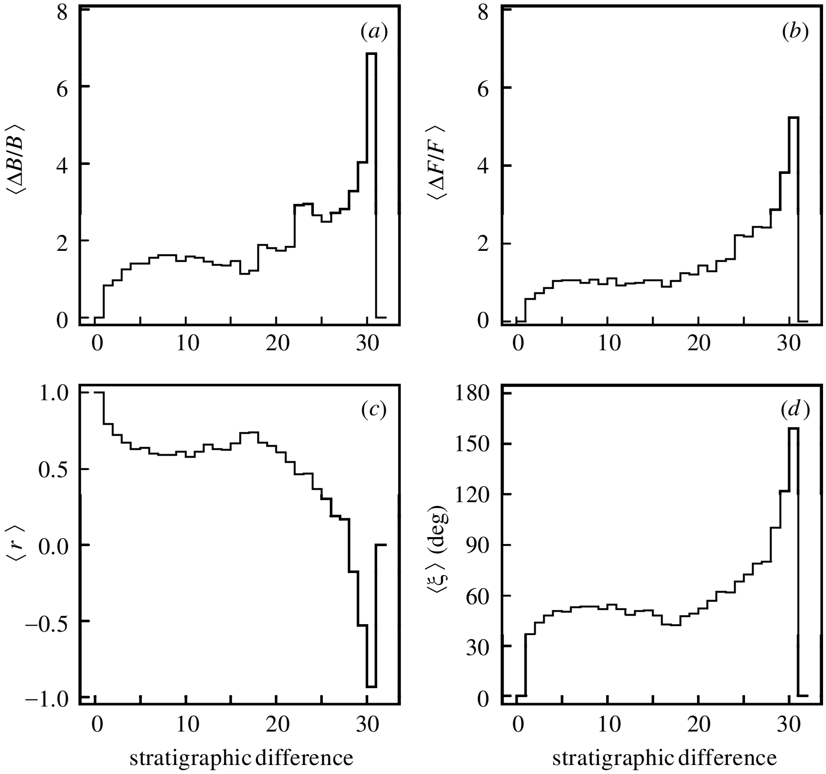

Figure 9. Mean correlations as a function of difference in stratigraphic position (height), where no directional cut-off has been applied, $\xi < 180^\circ$: (*a*) relative secular difference $\Delta B/B$, equation (4.2); (*b*) Relative intensity difference $\Delta F/F$, equation (4.4); (*c*) angular correlation r, equation (4.6); (*d*) angular difference ξ, equation (4.7).

cally correlated with the low in field intensity F, they are both in the Atlantic, we see in figure 1 that most of this correlation is due to the angular variation $\partial_t\mathcal{A}$; the relative intensity variation $\partial_t\mathcal{F}$ for the modern field is not so simply related to the mean intensity. In the same figure we also see that the angular correlation r and angular difference ξ are both useful measures of the angular variation; they both show a simple geographical correspondence with angular variation. Interestingly, what these maps teach us is that, since secular variation is such a strong function of geographical location, hypothetical consecutive lava depositions occurring 150 years apart in (say) the Indian Ocean, would record angular differences of only *ca.* 4°, while in the middle of the Atlantic Ocean they would record differences of *ca.* 40°. More generally, identical differences in successive depositional dates can correspond to a wide range of palaeovector differences.

Another hint of the complexity of secular variation comes directly from the palaeomagnetic data themselves, but because volcanoes erupt sporadically, palaeomagnetic lava data represent a temporally discontinuous record of geomagnetic secular variation. It is possible for multiple lava flows to be deposited over a duration of time that is short compared with the rate of secular variation, in which case the flows will preserve more or less similar records of the magnetic field. Across stratigraphically

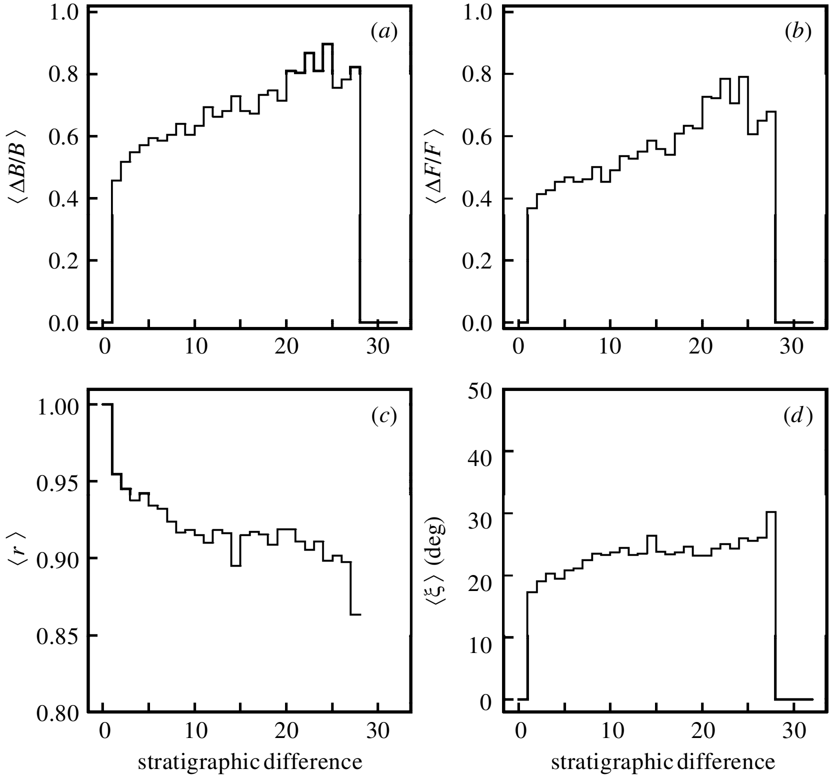

Figure 10. Mean correlations as a function of difference in stratigraphic position (height), where our preferred directional cut-off has been applied, $\xi < 60^\circ$: (*a*) relative secular difference $\Delta B/B$, equation (4.2); (*b*) relative intensity difference $\Delta F/F$, equation (4.4); (*c*) angular correlation r, equation (4.6); (*d*) angular difference ξ, equation (4.7). Note that the average correlations decrease (increase) with an increase (decrease) in stratigraphic positional difference. The vertical axes have different ranges than those in figure 9.

adjacent flows, how are angular differences correlated with intensity differences? In figure 11 we compare the difference in field direction between successive flows ξ versus the absolute difference in field intensity ΔF between (the same) successive flows. Since our database encompasses reversals and excursions, both difference in direction and difference in intensity between successive depositions can be large. However, notice that sometimes the direction changes by only a few degrees while the intensity changes by tens of microteslas, an amount comparable with the present surface intensity of the Earth's magnetic field. Given the present rate of westward drift in the Atlantic, it might be thought that relatively small differences in field direction represent only a few years of intervening time; on the other hand, given the rate of decay of the dipole, the intervening times between these successive depositions could be centuries. Similar interpretations can be made concerning the difference in field direction versus the relative difference in field intensity $\Delta F/F$ between successive flows, where we see no obvious relationship between the two quantities. As Love (1998) has remarked, differences in palaeovectors between successive flows reflect a broad range of differences in depositional times.

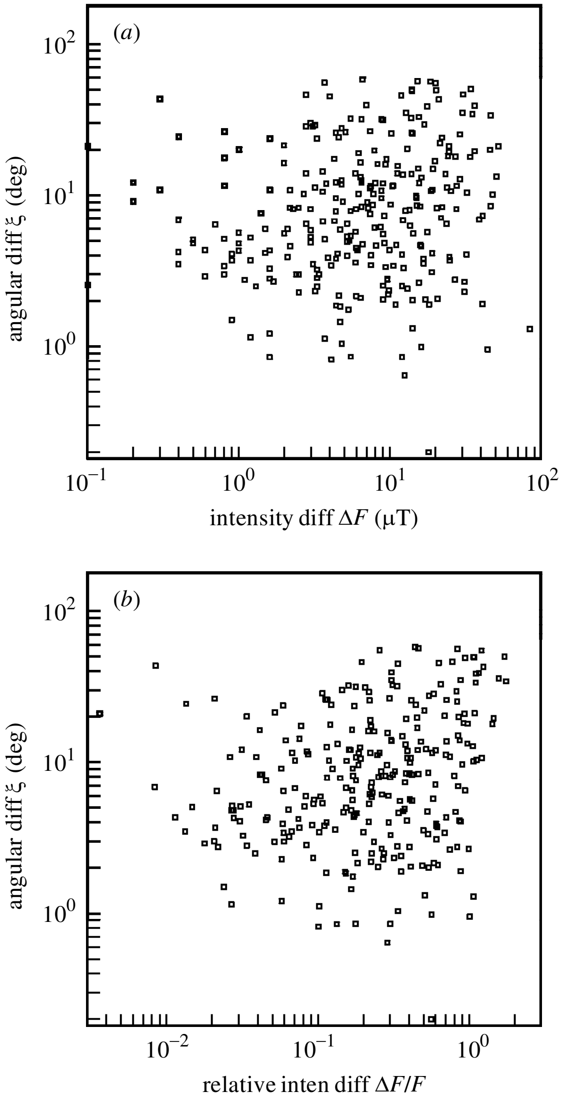

Figure 11. Angular difference ξ between stratigraphically adjacent lava flows versus: (a) intensity difference ΔF; (b) relative intensity difference $\Delta F/F$. The lack of correlation between angular differences and intensity differences demonstrates that the rate of secular variation is itself variable.

These observations have important implications for the analysis of palaeomagnetic data. With the temporal irregularity of volcanic activity in mind some investigators consider similar consecutive palaeomagnetic data to be redundant and they attempt to 'fix' the lava records by combining similar data taken from stratigraphically adjacent lava flows (McElhinny *et al*. 1996; Valet *et al*. 1999; Camps *et al*. 1999). Although it is not difficult to appreciate the motivation for such a treatment, it is important to recognize that it reflects a prejudice that secular variation is somehow temporally stationary. In fact, we do not *a priori* know the time dependence of

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the palaeomagnetic field; indeed, one of the reasons for studying palaeomagnetism is to discover the secular variation. Averaging similar palaeovectors, while at the same time leaving others unaveraged, defeats the purpose of the analysis. If averaging is done, it must be done objectively, not after subjective selection: exactly when are data considered to be similar and when are they considered to be different? Sometimes data are combined if the angular difference between the palaeovectors is less than the combined angular uncertainty (α_{95}) of the vectors. Yet we see clearly in both the modern field (figure 1) and in the palaeomagnetic data (figure 11) that secular variation is complex and is itself variable (like volcanic activity, secular variation might even be sporadic). Since successive lava flows with depositional dates differing by centuries can yield palaeomagnetic vectors that can range from either nearly identical to very different, the practice of selectively combining similar stratigraphically adjacent palaeovectors will result in a biased analysis and, in any case, does not solve the problems of uneven temporal sampling. Statistical analyses can be conducted, even on data which are not uniformly distributed in time, but not on data which have been partly averaged beforehand.

The fact that palaeomagnetic lava data are serially correlated is symptomatic of temporally continuous, but not necessarily stationary, variation of the magnetic field itself. The degree of correlation between individual data pairs depends on both the characteristic time-scale of the secular variation and the difference in depositional times. As such, correlations between individual palaeomagnetic data represent information about both secular variation and volcanic activity. Unfortunately, accurate radiometric dates represent the only potentially objective means of determining whether or not particular pairs of palaeovectors are similar because of nearly coincidental depositional dates or because of quiescence in secular variation, and such accurate dating is lacking for the vast majority of palaeomagnetic data. We can, however, address some of these difficulties by considering large collections of palaeomagnetic data. Since volcanic activity is unrelated to, and therefore uncorrelated with, variations in the magnetic field, then the durations between consecutive lava depositions, although by no means uniformly distributed in time, are at least unbiased with respect to geomagnetic activity. As a result, provided enough data are analysed and provided the analysis is appropriate, it should not be necessary to combine similar palaeovectors; instead we can use the mean of correlations between many serially ordered lava data to study the secular variation. The rate of secular variation is itself variable, and it is this variation of the variation and its possible functional dependence on intensity that we seek to investigate in the next section.

7. A single palaeomagnetic record: Steens Mountain

Before we consider the secular variation and intensity for the entire database, it is worthwhile inspecting a single record in some detail. Among the various sites, the most detailed individual lava record of a transition is the (15.5 Ma) reversal from Steens Mountain, Oregon (Mankinen *et al.* 1985; Prévot *et al.* 1985*a*). This record is known largely because of some puzzling data taken over the thickness of two specific flows (A41 and A42); because of thermal cooling after the initial deposition, data from different depths within the flows presumably represent different times. But since these data indicate extremely rapid fluctuations in field directions, as much as 6° per day, the analysis is controversial. For this reason we have omitted from our database

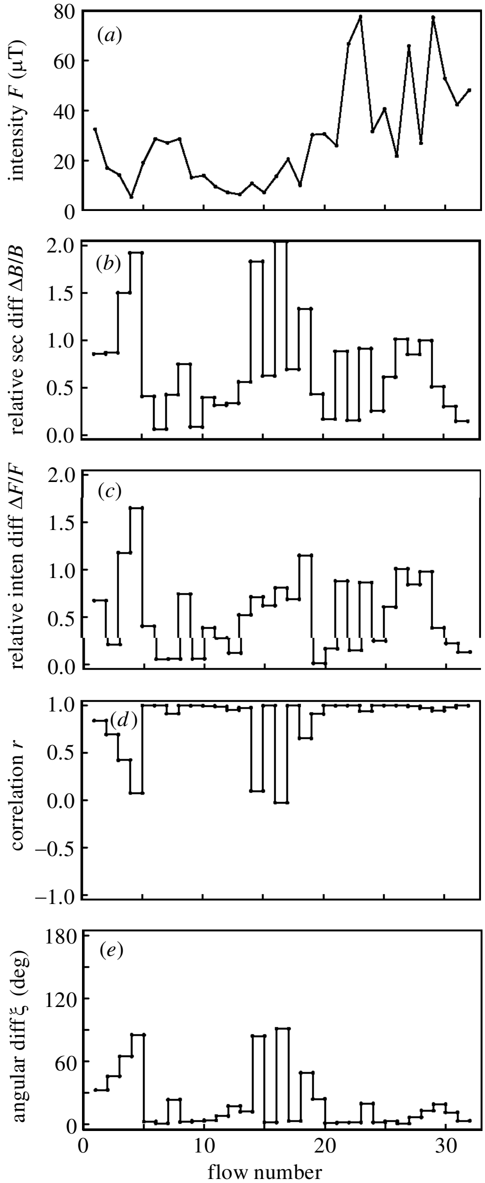

Figure 12. A comparison of intensity and correlations from the Steens Mountain data: (a) intensity F; (b) relative secular difference $\Delta B/B$, equation (4.2); (c) relative intensity difference $\Delta F/F$, equation (4.4); (d) angular correlation r, equation (4.6); (e) angular difference ξ, equation (4.7). Note that angular secular variation is, on average, largest (smallest) when the intensity is low (high).

the data from these two flows. The remaining Steens Mountain data, however, are generally considered to be reliable, and in figure 12 we show intensity along with the different measures of palaeomagnetic correlation $(\Delta B/B, \Delta F/F, r, \xi)$, each as a function of stratigraphic position. Because volcanoes erupt sporadically, the resulting palaeomagnetic records are often fragmentary. Certainly many of the abrupt differences in field intensity and direction between stratigraphically adjacent lava flows seen in the Steens data, and also in other volcanic data, are due to uneven temporal recording of the secular variation. Nonetheless, even in this single palaeomagnetic record we see some of the complex temporal dependence of the transitional field: at times the direction changes dramatically between successive lava depositions, while the intensity hardly changes at all; at other times the direction hardly changes, while the intensity changes considerably. Interestingly, from this record we also see that the direction tends to change most rapidly when the intensity is, on average, low, but relative intensity differences can be large even when the field is, on average, rather strong. In other words, at least for this single record, relative intensity differences are not well correlated with intensity, but mean angular differences might be inversely related to the intensity. With so few data, and lacking an absolute time-scale, more quantitative statements are difficult to make.

8. Secular variation and intensity: method

To inspect the data collectively for a possible relationship between secular variation and intensity we calculate the various measures of correlation $(\Delta B/B, \Delta F/F, r, \xi)$ for stratigraphically adjacent flows. We divide the range of intensity values F into intervals, each denoted k, and since each correlation depends on a pair of palaeovectors, we bin the various correlation measures once for F_j and once for F_{j+1}. Within each bin we then calculate the mean of the various measures of correlation. Since these means become independent of the number of data within each bin, N_k, as $N_k \to \infty$, then, provided there are enough data in each bin, we avoid sampling bias due to the over-representation of transitional low-intensity data in the literature. Moreover, since our database consists of data from serially ordered lava flows, and since volcanic eruptions and the durations between successive depositions are uncorrelated with variations in the magnetic field, there is no reason to expect that the correlations are a biased measure of secular variation.

We estimate the reliability of the mean correlations by a bootstrap analysis (Efron 1982). Let us denote by $\mathcal{C}$ the population of individual correlations within the kth bin. A population of test correlations, the nth being denoted $\mathcal{C}^n$, is generated by random sampling with replacement from $\mathcal{C}$. Because of the replacement the test correlation sets typically have some duplicate coefficients, and of course, are also missing a corresponding number of coefficients contained in the actual dataset. The dispersion of the test means gives a measure of the reliability of the mean correlation, something we express in terms of confidence limits.

9. Secular variation and intensity: results

In figure 13 we show relative secular difference $\Delta B/B$ as a function of local intensity F, from stratigraphically adjacent pairs of palaeovectors such that all $\xi < 60°$. The individual $\Delta B/B$ coefficients show a great deal of scatter, and the mean $\langle \Delta B/B \rangle$ does not show a particularly simple functional dependence on intensity. Relative secular difference shows a slight tendency to increase with decreasing intensity below *ca.* 50 μT, but the relative secular difference is actually slightly larger between 50 and 80 μT than it is between *ca.* 30 and 50 μT; there are relatively few palaeovectors with intensities greater than 80 μT. Given the lack of a simple trend and the size of

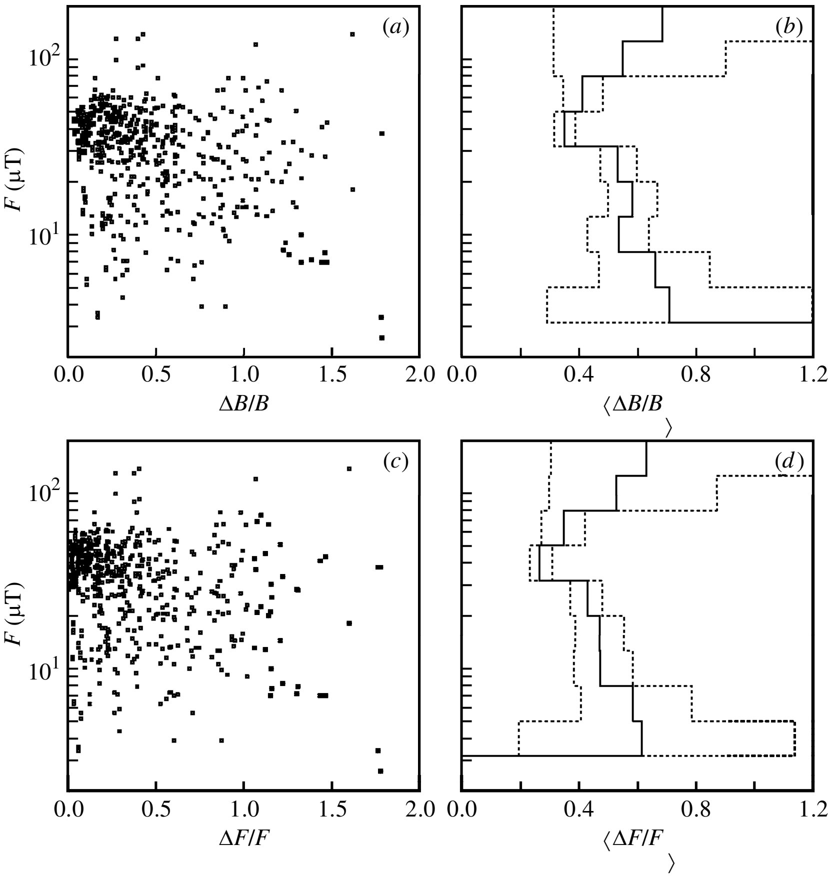

Figure 13. Correlations as a function of intensity F for an angular difference cut-off of 60°: (*a*) relative secular difference $\Delta B/B$, equation (4.2); (*b*) mean relative secular difference $h\Delta B/Bi$; (*c*) relative intensity difference $\Delta F/F$, equation (4.4); (*d*) mean relative intensity difference $h\Delta F/Fi$. The dashed lines represent the 95% confidence limits.

the confidence limits (shown at the 95% level), this result is, unfortunately, rather equivocal. Similar things may be said about the relative intensity differences $\Delta F/F$; the confidence limits are more restrictive here, and yet the lack of a clear trend requires that we be cautious about drawing firm conclusions. These mixed results are consistent with our inspection of the single record from Steens Mountain and, indeed, with our observations concerning relative intensity variation $\partial_t\mathcal{F}$ for the modern field, where a correlation with intensity is not obvious.

On the other hand, the angular secular variation is somewhat easier to interpret. In figure 14 we show the angular correlation coefficient r as a function of local intensity F, from stratigraphically adjacent pairs of palaeovectors such that all $\xi < 60°$. The individual correlation coefficients r show a great deal of scatter, but the mean $\langle r\rangle$ shows a fairly simple inverse functional dependence on intensity. Although the confidence limits are cause for sober caution, the trend is fairly clear: the correlation is highest (lowest) when and where the local intensity is highest (lowest). This result is similarly, though not independently, expressed by the angular difference ξ. We conclude that the available volcanic palaeomagnetic data appear to be

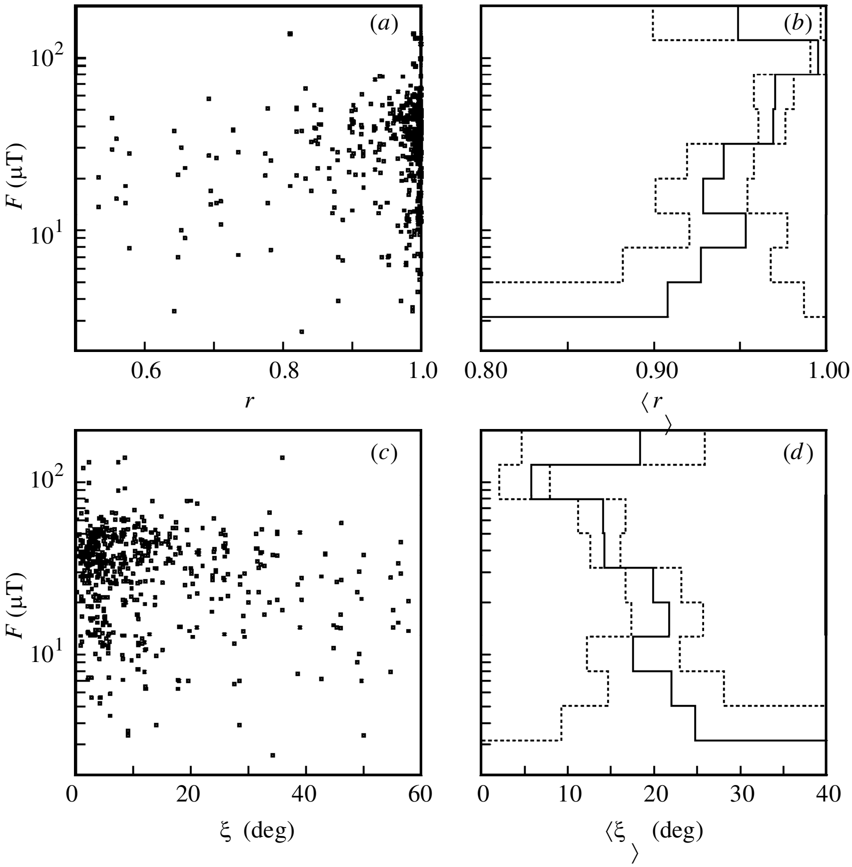

Figure 14. Correlations as a function of local intensity F for an angular difference cut-off of 60°: (*a*) angular correlation r, equation (4.6); (*b*) mean angular correlation $\langle r\rangle$; (*c*) angular difference ξ, equation (4.7); (*d*) mean angular difference $\langle\xi\rangle$. Note that the mean correlation $\langle r\rangle$ increases (decreases) with increasing (decreasing) intensity; the mean angular difference shows the opposite functional relationship with intensity. The lower limit on the $\langle r\rangle$-axis and the upper limit on the $\langle\xi\rangle$-axis are determined by the expected value for a random and isotropic distribution of directions given by equations (4.8) and (4.9); since the data tend to be correlated, the mean correlation always exceeds $\bar{r}(60°)$ and the mean angular difference is less than $\bar{\xi}(60°)$. The dashed lines represent the 95% confidence limits.

consistent with an enhancement (quiescence) of angular secular variation when the field intensity is low (high). These observations qualitatively are consistent with our inspection of the single record from Steens Mountain and, indeed, with our observations concerning angular secular variation $\partial_t \mathcal{A}$ for the modern field, where a correlation with intensity is clear. In a more quantitative respect, for the Earth's present average intensity (*ca.* 52 μT) and the present average rate of angular secular variation (*ca.* 4.4° per century), the corresponding average angular difference $\langle\xi\rangle$ from figure 14 of *ca.* 15° amounts to an average duration between successive depositions of about 340 years. From this figure we also see that the range of $\langle\xi\rangle$ indicates that secular variation may change by at least a factor of two over the range of possible field intensities.

The statistical significance of the correlations and angular differences can be assessed by comparing their means with the expected values for random and uniformly (isotropically) distributed directions given for the particular cut-off ξ_c, namely $\bar{\xi}$ from equation (4.8) and $\bar{r}$ from equation (4.9). In figure 14 the lower limit on the $\langle r\rangle$-axis is given by $\bar{r}(60°)$ and the upper limit on the $\langle\xi\rangle$-axis is given by $\bar{\xi}(60°)$. Rapid successive depositions mean that the data tend to be correlated, with the mean correlation between stratigraphically adjacent flows always exceeding that expected for random isotropic data. In fact, the correlations can be said to be significantly different from a random and isotropic distribution of directions with better than 95% confidence, since the mean correlation minus the 95% confidence limit exceeds $\bar{r}(60°)$, and the mean angular difference plus the 95% confidence limit is less than $\bar{\xi}(60°)$.

The reader may be curious about the sensitivity of our analysis to the angular-difference cut-off ξ_c, and therefore in figure 15 we show the mean correlation $\langle r\rangle$ as a function of intensity for a variety of different cut-offs ξ_c. From this figure we see that, regardless of the details of the angular-difference cut-off, the inverse relationship is still evident: the correlation is highest (lowest) when and where the local field is strongest (weakest). In this figure the lower limits on the $\langle r\rangle$-axes are given by $\bar{r}(\xi_c)$, and as with figure 14, the statistical significance of the correlations is established by noting that the mean correlation minus the 95% confidence limit exceeds the correlation for random isotropic data $\bar{r}(\xi_c)$. Obviously, angular differences larger than (say) 90° probably represent long hiatuses in lava deposition, and we feel that our adoption of a 60° cut-off throughout most of this analysis represents a reasonable compromise value for rough estimation of the secular variation. The scatter in $\langle r\rangle$ above 80 μT from one cut-off to another is due to the few data determining the mean correlation for such high intensities, a fact emphasized by the larger confidence limits.

Finally, it might be suggested that the trend we see in correlations between stratigraphically adjacent lava flows could be due to scatter in the data introduced by the inability of lavas to preserve a sufficiently accurate record of the palaeofield during periods of low intensity. To check this, in figure 16 we show the mean relative intensity error $\langle\sigma(F)/F\rangle$ and the mean directional error $\langle\alpha_{95}\rangle$ each as a function of intensity. Although the relative intensity errors show a dependence on intensity, they are largest (smallest) when the intensity is low (high), they are about an order of magnitude less than the mean relative intensity differences seen in figure 13. In other words the signal-to-error ratio is sufficiently high that we can conclude that errors on individual data are not a significant factor in our study of relative intensity variation. Concerning the directional errors, they show almost no dependence on intensity, except for the lowest intensity values where the number of data are actually few. A more formal statistical treatment of directional errors and their affect on a related investigation of the anisotropy of secular variation can be found in Love (2000). As with relative intensity variation, however, we can conclude that the errors on individual directions are not a significant factor in our study of angular secular variation (figure 14).

10. Conclusions

Clearly studies such as ours will benefit from the continued collection of full vectorial palaeomagnetic data from stratigraphic sections of lava. Although we have not

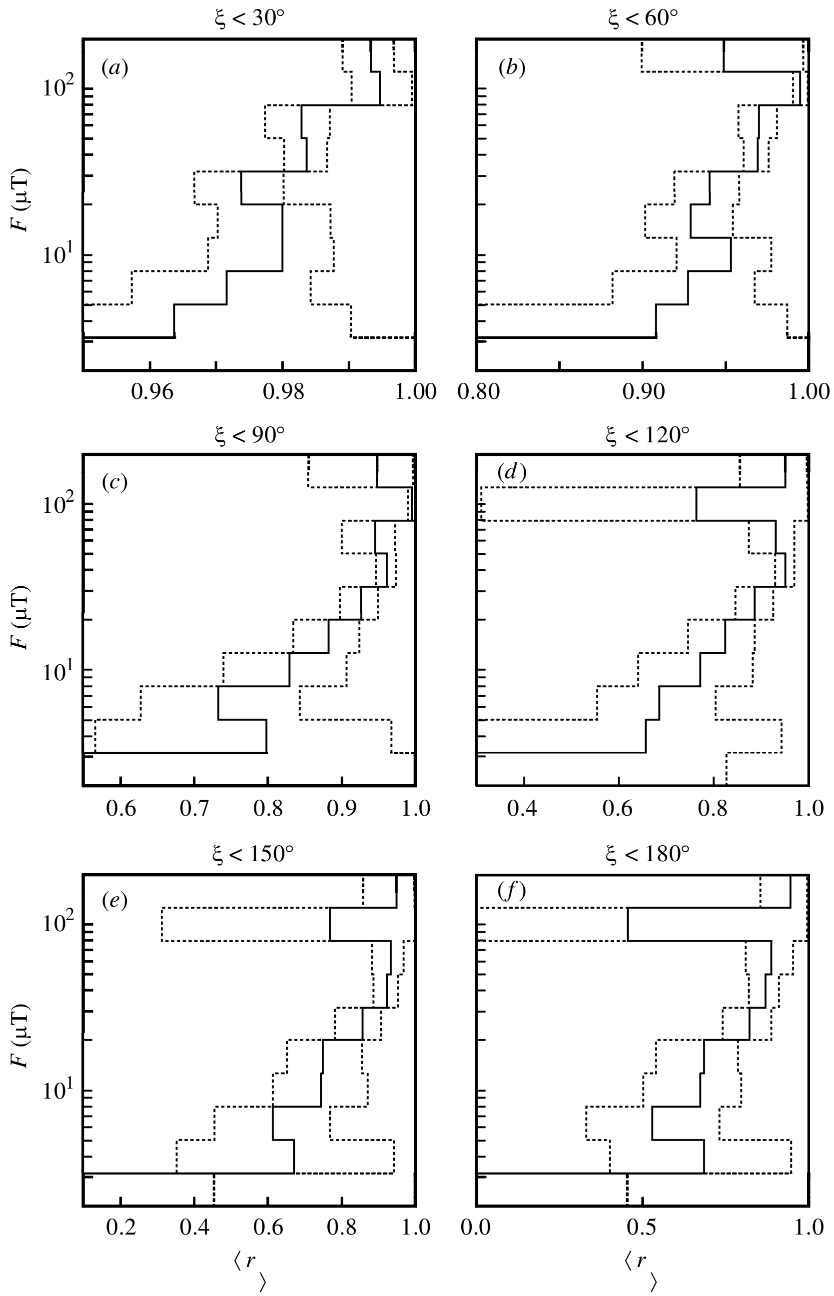

Figure 15. Mean angular correlations $\langle r\rangle$ as a function of local intensity F for a variety of angular-difference cut-offs ξ_c: (*a*) 30°; (*b*) 60°; (*c*) 90°; (*d*) 120°; (*e*) 150°; (*f*) 180°. The lower limits on the $\langle r\rangle$-axes are determined by the expected angular correlation for a random and isotropic distribution of directions given for the cut-off ξ_c, namely $\bar{r}$ from equation (4.9); since the data tend to be correlated, the mean correlation always exceeds $\bar{r}(\xi_c)$. The dashed lines represent the 95% confidence limits.

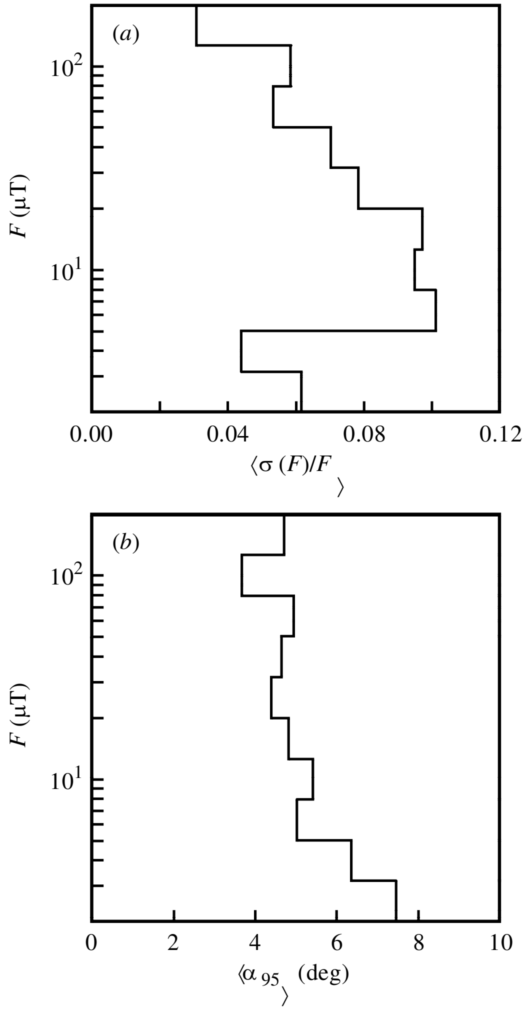

Figure 16. Mean errors as a function of intensity: (*a*) mean relative intensity error $h\sigma(F)/Fi$; (*b*) mean directional precision parameter $h\alpha_{95}i$. These mean errors are relatively small compared with the intensity and angular differences considered in this analysis.

been able to draw a firm conclusion concerning relative intensity variation, serial correlations of the available data are indicative of a systematic relationship between angular secular variation and intensity: angular secular variation is enhanced (quiescent) when field intensity is low (high). In other words, angular secular variation is apparently inversely related to local field strength. Insofar as polarity transitions are coincident with periods of weak field intensity, our conclusion here is qualitatively consistent with that of Love (2000), who inspected a large number of directional lava data, with Aundunsson & Levi (1997), who examined a thick layer of slowly cooling basalt, and with (Valet *et al*. 1986), who examined sedimentary data. We also note that (Glatzmaier & Roberts 1995) found an enhancement of secular variation during their numerical simulation of a reversal.

In the future, one way to incorporate the information content of serially correlated volcanic data into statistical modelling would be to represent the volcanic record of secular variation by some sort of Markov process (see, for example, Constable 1990). Ideally one would also incorporate the information content provided by (the few) radiometric dates, along with allowance for their associated errors. Such a programme would at least be an improvement over current approaches. Of course, that volcanic palaeomagnetic data are not statistically independent, but are often correlated, is symptomatic of continuous secular variation being sampled randomly and discontinuously with the deposition of each lava flow. In other words, statistical models are not so much descriptions of the Earth's magnetic field, but rather descriptions of the combined magnetic and volcanic system.

We finish by noting some of the implications of this work for theoretical modelling of the dynamo. It is sometimes suggested that rapid changes in the Earth's magnetic field, for example during reversals, are associated with increased convective activity in the core (Prévot *et al*. 1985*b*; Valet *et al*. 1986). This idea seems plausible at first, but is not especially consistent with what we know about rotating magneto-convection. Linear stability theory tells us that the presence of a magnetic field in a rotating fluid system actually enhances convection, since part of the Lorentz force offsets the restrictive Coriolis force (Chandrasekhar 1961); there is evidence that this also applies in the fully convective (nonlinear) regime (Roberts 1978; Stevenson 1979; Fautrelle & Childress 1982). Given these theories, it might be thought that the field should actually change more rapidly when the field strength is high, a contradiction of the observations presented here. Many aspects of dynamo theory have yet to be resolved, and the actual mechanics of the core are certainly complex, but we can suggest that perhaps highly vigorous convection in the core (accompanied by increased field strength) is not as time dependent as less vigorous convection (accompanied by decreased field strength). This possibility appears to be consistent with theoretical work by Hollerbach & Jones (1993), who found that the electrically conducting inner core stabilizes the magnetic field; presumably the Lorentz coupling between the solid inner core and the convecting outer core is enhanced (lessened) with increased (decreased) field strength, thus accounting for the inverse relationship between secular variation and field strength found here.

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